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Real Time Data Association for FastSLAM

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***Abstract*— The ability to simultaneously localise a robot and accurately map its surroundings is considered by many to be a key prerequisite of truly autonomous robots. This paper presents a real-world implementation of FastSLAM, an algorithm that recursively estimates the full posterior distribution of both robot pose and landmark locations. In particular, we present an extension to FastSLAM that addresses the data association problem using a nearest neighbour technique. Building on this, we also present a novel multiple hypothesis tracking implementation (MHT) to handle uncertainty in the data association. Finally an extension to the multi-robot case is introduced. Our algorithm has been run successfully using a number of data sets obtained in outdoor environments. Experimental results are presented that demonstrate the performance of the algorithms when compared with standard Kalman Filter-based approaches.**

I. Introduction

Simultaneous localisation and mapping (SLAM) is the process that enables a mobile robot to localise and build a map of an unknown environment using only observations relative to the most relevant features detected by its sensors. The solution to the SLAM problem is considered by many as a key prerequisite for making a robot fully autonomous [11], [17]. This problem has been the object of significant research in the last decade.

The classical SLAM solution was introduced in a seminal paper by Smith, Self and Cheeseman [16]. The authors proposed the use of an Extended Kalman Filter (EKF) to consider uncertainties in vehicle pose and the map, as both are acquired from potentially noisy sensor measurements. This approach has been widely adopted and initiated an important line of research in SLAM. There have been several implementations of the EKF SLAM in different environments, such as indoors [10], underwater [22] and outdoors [6].

One of the main problems of the SLAM has been the computational requirement. The complexity of SLAM can be reduced to $O(N^2)$, where N is the number of state variables needed to represent the landmark positions and the robot pose. Further optimisations and simplifications have been proposed to solve the computational problem using EKF approaches in very large environments [7], [14], [21], [8]. The EKF filter assumes Gaussian noise and requires a linearization of the robot and sensor models. These assumptions can be problematic when the distributions are not unimodal, or when working with large uncertainties and strong non-linearities.

Optimal SLAM techniques based on Bayesian filtering with non-Gaussian assumptions can be extremely expensive due to the high dimensionality of the map representation. This makes the full Bayesian solution difficult to apply in real time. In a recent publication, a novel algorithm named FastSLAM was presented which addressed the real time implementation of the SLAM problem from a Bayesian point of view [12]. The authors used a particle filter [4], [5] to estimate the vehicle pose and EKF filters to estimate the location of features in the map. This algorithm partitions SLAM into a localisation and a mapping problem. FastSLAM uses a modified particle filter to estimate the robot pose. Each particle has an independent EKF running for each landmark in the map to estimate its position. However, the original paper presents results assuming the data association is known, and no sound solution has been provided when data association is unknown.

This paper attempts to fill this gap. The data association is arguably one of the most challenging problems in SLAM: As the robot navigates, it has to determine whether different sensor measurements correspond to the same feature in the world. False data associations induce catastrophic failures on EKF-style SLAM solutions [3]. To overcome this problem, we extend the original algorithm addressing the data association problem with different levels of complexity by using standard chi-square test and Nearest Neighbour approaches. Experimental results and comparison of EKF SLAM and FastSLAM are presented. Also initial results in a centralised multi-robot SLAM are presented.

II. Bayesian Estimation and FastSLAM

To introduce FastSLAM, it is worthwhile to present the SLAM problem from a a probabilistic point of view. SLAM algorithms commonly compute the probability distribution

$$P(x_k, m | Z^k, U^k, x_0) \tag{1}$$

where x_k is the robot pose vector at time k, m is the map state vector, Z^k is the set of observations received until time k, U^k are the control inputs and x_0 the initial position [18]. This equation describes the joint posterior density of the robot pose and map at time k, given the initial robot pose, and all the observations and control inputs up to time k.

In probabilistic terms, the SLAM problem is a Markov process. This means that the state at time $k-1$ embodies all the necessary information to propagate the system states to time k. This Markov property is at the core of all probabilistic approaches to the SLAM problem. In particular, robot motion is usually considered a Markov process. Its probability distribution is described as

$$P(x_k|x_{k-1}, u_k) \tag{2}$$

Moreover, the observation model, which relates observations to the state of the world, is described by a probabilistic model that adheres to the Markov property:

$$P(z_k|x_k, m) \tag{3}$$

SLAM is then achieved by applying Bayes filtering to solve (1) for all k, using (2) and (3). The resulting filter involves a convolution (for the prediction step) and a multiplication (for the measurement update):

$$\begin{aligned} &P(x_k, m|Z^k, U^k, x_0) \\ &\quad \propto \quad P(z_k|x_k, m)P(x_k, m|Z^{k-1}, U^k, x_0) \end{aligned} \tag{4}$$

with

$$\begin{aligned} &P(x_k, m|Z^{k-1}, U^k, x_0) \\ &= \int P(x_k|x_{k-1}, u_k)P(x_{k-1}, m|Z^{k-1}, U^{k-1}, x_0)dx_{k-1} \end{aligned} \tag{5}$$

Here z_k denotes the measurement at time k, and u_k the controls asserted between x_{k-1} and x_k.

The standard Bayesian solution (4) and (5) can be extremely expensive for high-dimensional maps m. The EKF approach presents a tractable solution to this problem assuming that the noise is Gaussian and independent over time. Furthermore, the EKF solution is approximate in that it linearises the motion and observation models, although this is not an issue in many practical applications. One fundamental problem is the fragility of these EKF methods under incorrect data associations. These methods tends to fail catastrophically in such situations since they can only handle unimodal distributions [3].

FastSLAM is an efficient algorithm for the SLAM problem that is based on a straightforward factorisation. The factorisation is illustrated in Figure 1 (image reprinted with permission from [12]). It is based on the observation that if one knew the true path of the robot, the individual landmark localisation problems are mutually independent. In practice, the path is of course unknown. However,

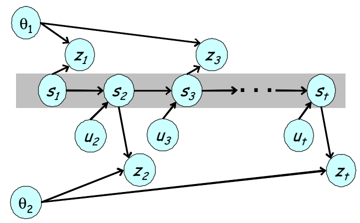

Fig. 1. The SLAM problem: The robot moves from pose x_1 through a sequence of controls, $u_1, u_2, \ldots$. As it moves, it measures nearby landmarks. At time $k = 1$, it observes landmark m_1 out of two landmarks, $\{m_1 \ and \ m_2\}$. The measurement is denoted z_1 (range and bearing). At time $k = 2$, it observes the other landmark, m_2, and at time $k = 3$, it observes m_1 again. The resulting landmark estimation problems are conditionally independent given the robot's path. The gray shading illustrates a conditional independence relation.

the conditional independence enables us to estimate the posterior (1) in the following factored form:

$$\begin{aligned} &P(x^k, m|Z^k, U^k, x_0) \\ &\quad = \quad P(m|x^k, Z^k, U^k, x_0)P(x^k|Z^k, U^k, x_0) \\ &\quad = \quad \prod_{i=1}^{N} P(m_i|x^k, Z^k, U^k, x_0)P(x^k|Z^k, U^k, x_0) \end{aligned} \tag{6}$$

This factorisation is the fundamental idea behind FastSLAM: FastSLAM decomposes the SLAM problem into a localisation and N landmark positions estimation problem. Furthermore, FastSLAM relies on a particle filter to estimate the robot posterior

$$P(x^k|Z^k, U^k, x_0) \tag{7}$$

This particle filter can be updated in constant time for each particle in the filter. It furthermore relies on N *independent* Kalman filters for the N landmark estimates

$$P(m_i|x^k, Z^k, U^k, x_0) \tag{8}$$

As shown in [12], the entire filter can be updated in time logarithmic in the number of landmarks N. While other solutions of similar efficiency exist [7], [14], [9], FastSLAM can also handle non-linear robot motion models.

III. Data Association

The original FastSLAM paper does not address the important issue of data association. In every SLAM problem, the measurements need to be associated with the underlying states that are being observed. This is usually referred as the *data association problem* and is probably one of the most difficult problems in SLAM or localisation applications. Successful data association involves association of the correct measurement with the correct state, initialising new tracks and detecting and rejecting spurious measurements. There are different ways to implement the data association process [1]. Most existing techniques are based on the innovation sequence and its predicted covariance. The innovation sequence ν_k

relates observations z_k to the underlying state estimates. In particular, it is defined as the sequence of differences between the observation z_k and the predicted observation based on the observation model and the predicted states $\hat{z}_k$

$$\nu_k = z_k - \hat{z}_k \tag{9}$$

The normalised innovation (or normalised distance) is defined as

$$d_k^2 = \nu_k^T S_k^{-1} \nu_k \tag{10}$$

Here S_k is the innovation covariance matrix. It is well known that if the innovations ν_k have a Gaussian distribution, then $\nu_k \nu_k^T$ will have a χ^2 distribution. The innovation sequence is the basis of the "gate validation" technique, which accepts the observations that are inside a fixed region of a χ^2 distribution, and rejects the observations that make the innovation fall outside these bounds. This determination is achieved by comparing the scalar obtained in (10) with a threshold value that is determined by fixing the region of acceptance of the χ^2 distribution (e.g.95%).

One of the most common data association techniques used in the SLAM literature is based on just this insight, in that it relies on the so-called *nearest neighbour filter* (NNF). The NNF uses the gate validation test to initially determine which landmark states are valid candidates to update with the observation received. Among all the possible landmarks, the one that is *nearest* to the observation is selected. All the other possible hypotheses are ignored. The NNF can fail to recover the true data association when validation gates overlap, and the observations fall within this overlapped region. In such cases, the NNF can associate the observation with an incorrect state. Such false data associations are known to induce catastrophic failures to the SLAM problem. When multiple observations are jointly processed a more robust data association is obtained. It is in fact a gate validation technique applied to a multi-dimensional observation. This joint data association intrinsically considers the geometric relation between a set of landmarks [19], [21]. A wider technique is presented in [20]. This technique creates a local map after a short time local SLAM, and matches this with a global map based on that vector data association.

In FastSLAM, the association can be determined on a per-particle basis, and hence different particles can be associated with different landmarks. Each particle may even have a different number of landmarks in their respective maps. This characteristic gives the filter the possibility of dealing with a multi-hypothesis association problem. *Multiple Hypothesis Tracking* algorithms (MHT) [15] maintain different tracks for each possible hypothesis of each observation. FastSLAM, when association decisions

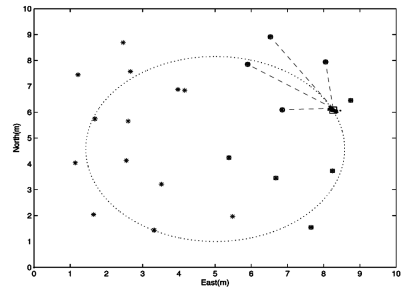

Fig. 2. Simulation environment: '*' actual landmark positions, '.' robot trajectory

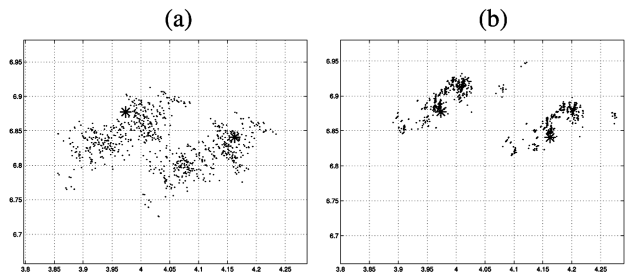

Fig. 3. Sequence of beacon positions estimation. '.' Gaussian means, '*' actual landmark positions. (a) Result obtained in the first lap (b) After three laps.

are made on a per-particle basis, are efficient instantiations of the MHT idea.

Our approach is based on this insight. As in the NNF, a gate validation test is used initially to determine the possible landmarks to be associated with the observation. The multiple hypothesis filter maintains a separate track for each hypothesis. Our MHT implementation using FastSLAM is put into effect by creating a new particle for each hypothesis. Each particle is split into $n+2$ particles, one for each of the n hypotheses, plus one particle for the non-association hypothesis (for spurious measurements) and one for the new landmark hypothesis. Eventually the resampling stage (a step in the particle filter) will eliminate the wrong hypothesis when subsequent observations are incorporated. Thus, our approach implements FastSLAM using particle-specific data association. This approach is, thus, fundamentally different from EKF approaches, where the implementation of MHT filters requires spawning parallel filters for the different hypotheses and using pruning techniques to discard the wrong ones. In FastSLAM, particles with wrong data association are (in expectation) more likely to disappear in the resampling stage than those that "guess" the right data association. In many ways, our approach is the natural extension of FastSLAM to deal with a SLAM problem with unknown data association.

Figure 2 illustrates the basic principle of FastSLAM using MHT data association in a simulation result. In this case we have two beacons at coordinates (4,6.8) separated less than 20 cm. The error in the sensor range was set to 15 cm. These parameters are very similar to the noise present in the outdoor experiment discussed further below. Figure 3 shows the landmarks' position estimation. The plot presents the landmarks' Gaussian means for all the particles, in (a) for the first lap and in (b) after the third lap. It can clearly be seen how the MHT implementation solves the problem of the ambiguity association: The surviving particles not only approximate the robot path well, but also the sequence of data associations along the way.

IV. Multi-Robot FastSLAM

The implementation was also extended to handle Multi-Robot SLAM problems. Multi-Robot SLAM involves a team of robots, which cooperatively acquire a single map of a shared environment. The multi-robot SLAM problem has been addressed using EKF approaches [13], [20]. Multi-robot SLAM with FastSLAM makes it possible to build a joint map even if the vehicles do not know their initial relative pose, although the number of particles needed will be dependent on this uncertainty.

There are two relevant strategies: Centralised and Decentralised Multi-Robot SLAM. From a particle filter perspective, localising one vehicle in the map of another is basically a localisation problem, which can be solved using the *Monte Carlo localisation* (MCL) algorithm [2]. This algorithm has been successfully applied to perform *global mobile robot localisation*, which addresses the problem of estimating a robot location from scratch in a known map. FastSLAM, thanks to its use of particle filters for robot pose estimation, can be viewed as a version of this approach applied to the more complex SLAM problem. It inherits from MCL the ability to globally localise a robot within a known map. In the context of multi-vehicle SLAM, the localisation takes place in the map acquired by a different robot.

V. Experimental Results

The algorithm presented was tested in two different outdoor environments. The first test environment was the top level of the car park building of the university campus. The full data set and the documentation is available at [23]. This testing site was chosen to maximise the number of satellites in view to obtain high quality GPS information. A kinematic GPS system of 2 cm CEP accuracy was used to evaluate the ground truth. A standard utility vehicle was fitted with dead reckoning sensors and a laser range sensor as shown in Figure 4. In this experiment, artificial landmarks were used that consisted of 60 mm steel poles covered with reflective tape. With this approach the feature extraction becomes trivial and the landmark observation model very accurate. Since the true position of the landmarks was also obtained with GPS, a true navigation map was available for comparison purposes. The second experimental run was done in a larger area with mild

Fig. 4. Victoria Park. The utility car used for this experiment is equipped with a Sick laser range and bearing sensor, linear variable differential transformer sensor for the steering and back wheel velocity encoder.

uneven terrain and different types of surfaces. The nature of the terrain created additional errors in the vehicle prediction, since wheel sleep and attitude errors are not taken into account in the prediction models of our current implementation. In our experiments, the most common relevant features in the environment were trees. The profile of trees was extracted from the laser raw data, and the most likely centre of the trunk was estimated using a geometric analysis of the range measurements. A set of individual KFs were implemented to reduce the errors due to the different profiles obtained when observing the trunks of the trees from different locations [6]. Nevertheless, feature detection and feature location were significantly less accurate than in our experiment involving artificial landmarks. Although the vehicle was also equipped with a kinematic GPS, this sensor was not accurate enough due to poor satellite availability, as expected in this type of environment. Nevertheless the information gathered was good enough to verify that actual errors were consistent with those estimated by the filter.

The next sections present experimental results with known and unknown data associations. The objective of the experiments with known data association is to compare FastSLAM with the standard EKF SLAM, and then separate the problem of filter consistency with the data association problem.

A. SLAM with Known Data Association

In our first series of experiments, the correspondences between the observation and the landmarks were assumed to be known. Instead of providing this information manually, we used a well tuned EKF to provide the correct data association for each measurement. The EKF algorithm was run with the same data set, dropping all measurements that were not associated with any landmark. As a result, FastSLAM only received measurements corresponding to actual landmarks in the environment. Figure 5 shows

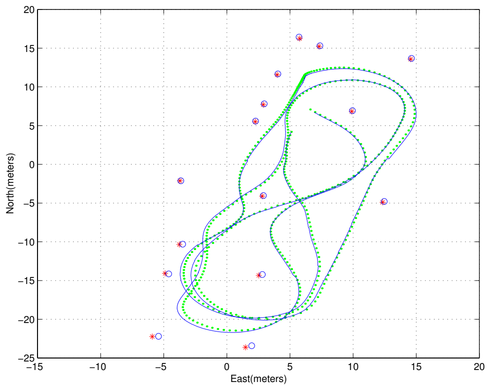

Fig. 5. Estimated path and landmarks with FastSLAM. The '-' is the path estimated, the '*' are the estimated beacon positions, the '.' is the GPS path reference and the 'o' are the beacon positions given by the GPS.

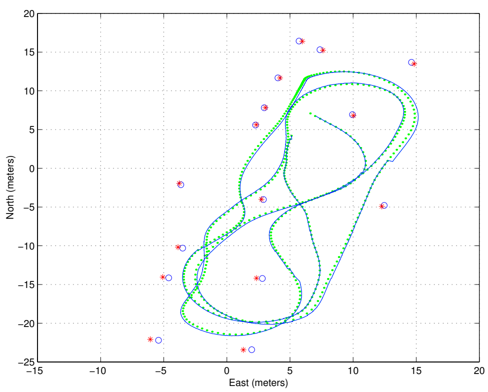

Fig. 6. Estimated path and landmarks with Kalman Filter. The '-' is the path estimated, the '*' are the estimated beacon positions , the '.' is the GPS path reference and the 'o' are the beacon positions given by the GPS.

(a) (b)

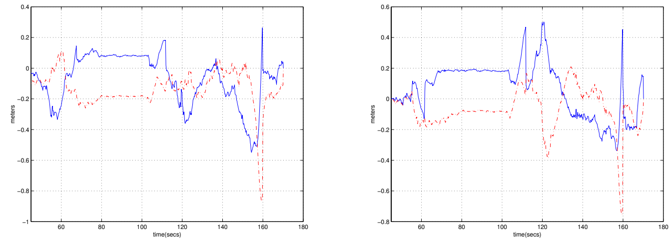

Fig. 7. SLAM error (a) FastSLAM position error in respect to the GPS position. '-' indicates the error in the East and '-.'in the North coordinates (b) EKF position error in respect to the GPS position. '-' indicates the error in East and '-.' in the North.

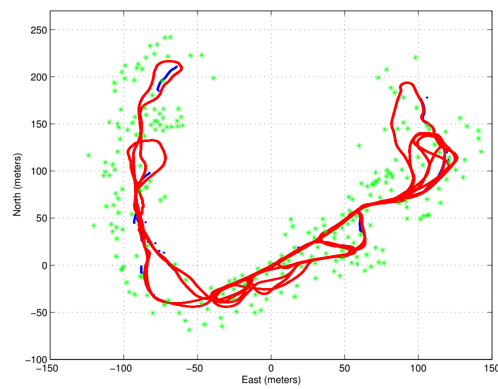

Fig. 8. Estimated path and landmarks in Victoria park. The '*' are the tree positions and the darker line is the GPS where position was reported.

the path and beacons' position estimation for the car park experimental run using the FastSLAM algorithm. This figure shows the particle average for the vehicle trajectory and the average of all the Gaussian means for the landmark locations. Figure 6 shows similar results obtained with the EKF based algorithm. Figure 7 presents the vehicle position error for the EKF and FastSLAM filter respectively. It should be noted that the error is very small and similar in magnitude and shape when compared with the GPS ground truth. This is important to verify the consistency of the FastSLAM algorithm.

The trajectories and map for the second experimental run for the FastSLAM and EKF are presented in Figure 8 and Figure 9 respectively. It was not possible to obtain full error plots for the trajectory since GPS is not available for most of the path. Nevertheless, the results compare very well with the full EKF SLAM results. It is important to remark that the filter operates for more than 4000 metres in this 20 minutes run. Figure 10 shows the superposition of the trajectory obtained with an outdated satellite image of the same environment. This satellite image was several years old at the time of the data collection; the vegetation is actually denser. The GPS was available only in a few areas, as shown in Figure 8 and Figure 9.

B. SLAM with Unknown Data Association

The next sections present the results of FastSLAM with unknown data association. Two different data association techniques are presented, one based on the nearest neighbour algorithm and the other on the multi hypothesis approach.

1) Nearest Neighbour Filter: This section presents the results of FastSLAM with unknown data association for the NNF technique. Figure 11 shows the map and the trajectory for the car park data set. In this case, the data association is an integral part of the algorithm, and FastSLAM is able to use all the available information. This feature is of fundamental importance when working

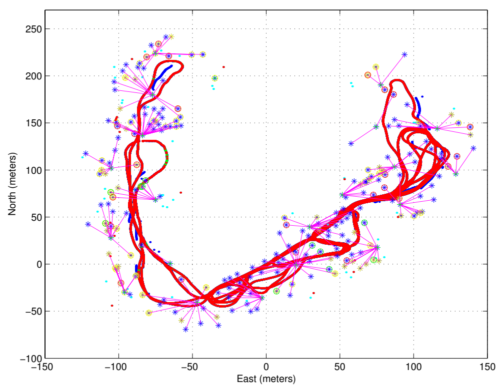

Fig. 9. Estimated path and landmarks using CEKF.

Fig. 10. Satellite picture of the park, with the estimated trajectory.

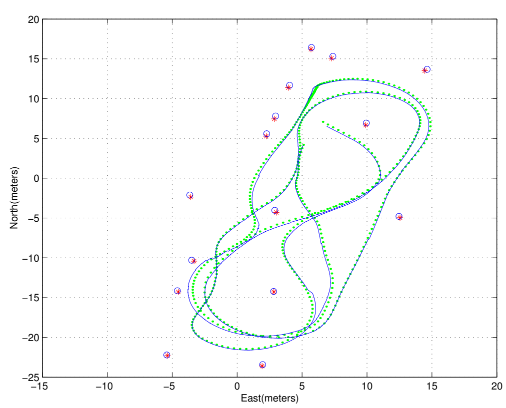

Fig. 11. Estimated Path and Landmarks with unknown data association. The '-' is the estimated path, the '*' are the estimated beacon positions, the '.' is the GPS path reference and the 'o' are the beacon positions given by the GPS.

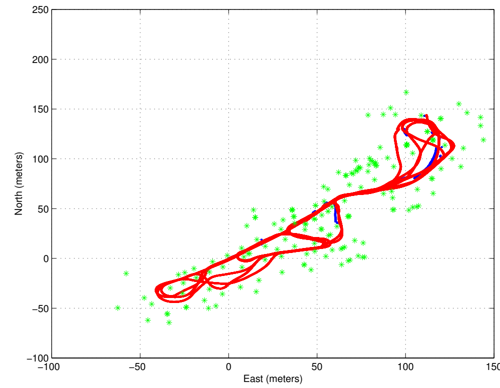

Fig. 12. Estimated Path and landmarks in Victoria park with unknown data association. The '*' are the tree positions and the darker line is the GPS where position was reported.

with low frequency sensors, where features remain within a field of view of the sensor only for short periods of time.

Figure 12 presents the results obtained for the more challenging Victoria Park data set. In this case, 800 sec of the trajectory and map are presented, using 200 particles. The results are very similar to the ones presented before with known data association. This illustrates that the approach is highly effective in handling data association problems.

2) Multi Hypothesis Implementation: Figure 13 presents the experimental results obtained using the MHT algorithm for data association. In this experiment, four beacons were located between 1 and 1.5 metres apart. The final landmarks' position estimation error was approximately 15 cm. The filter was run with 200 particles. As described before, an additional particle is added for each hypothesis in case of uncertainty in the association process. When the gate validation test indicates that more than one landmark can be associated with the actual observation, the algorithm creates new particles with each possible hypothesis. The set of hypotheses considers all possible landmark/observation combinations, plus the hypothesis for a new landmark. Finally, after a fixed number of observations, a resampling with a uniform distribution is implemented, reducing the number of particles to the original number.

C. Multi-Robot Experiment

A centralised implementation of Multi-Robot Fast-SLAM is presented in this section. With this approach the robots are required to send the information acquired, in this case the observation vector, to a central robot that runs the centralised filter. The simulation environment was basically the same as in Figure 2 except for the inclusion of two robots and a larger operating area. Figure 14

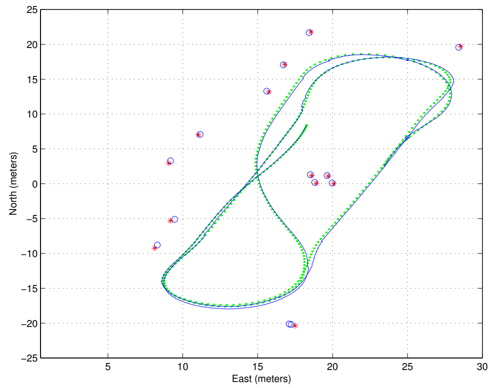

Fig. 13. Estimated Path and Landmarks, now with MHT data association. The '-' is the estimated path, the '*' are the estimated beacon positions, the '.' is the GPS path reference and the 'o' are the beacon positions given by the GPS.

(a) (b)

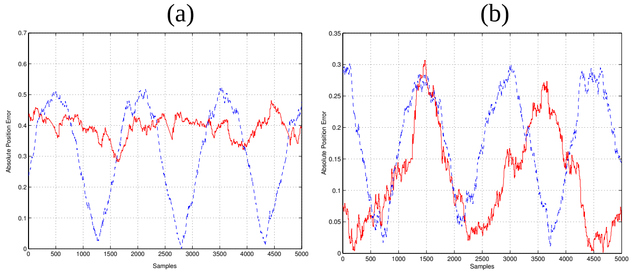

Fig. 14. Robots absolute position error. (a) Robot 1 ('-') and Robot 2 ('-.') working independently. (b) Multi-robot SLAM.

shows the position error in both robots, in (a) working independently and in (b) using the Multi-Robot FastSLAM algorithm. It can be seen that the overall error is smaller in the centralised algorithm. The filter was run with 500 particles when the robots were working independently and with 800 particles for the multi-robot case.

VI. Conclusion

This paper presented extensions to the FastSlam algorithm to implement data association. The experimental results performed in a variety of outdoor environments demonstrated the robustness of the algorithms and the ability to handle multiple hypotheses in a very efficient and elegant manner. Initial results with a centralised multi-robot FastSLAM implementation were also presented. Among the areas of current and future research are:

- use of the algorithm in the bearing-only problem using non-gaussian representations for the map
- implementation of a hybrid SLAM architecture, EKF SLAM-FastSLAM
- extension of the current algorithm to implement a decentralised multi-robot SLAM

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