Solving Computational and Memory Requirements of Feature Based Simultaneous Localization and Map Building Algorithms

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Abstract

This paper presents new algorithms to implement simultaneous localisation and map building (SLAM) in environments with very large number of features. The algorithms present an efficient solution to the full update required by the Compressed Extended Kalman Filter algorithm (CEKF). It makes uses of the Relative Landmark Representation (RLR) to develop very close to optimal de-correlation solutions. With this approach the memory and computational requirements are reduced from $\sim O(N^2)$ to $\sim O(N*N_b)$, being N and N_b proportional to the number of features in the map and features close to the vehicle respectively. Experimental results are presented to verify the operation of the system when working in large outdoor environments.

1 Introduction

Autonomous navigation in outdoor environments present significant challenges due to the lack of reliable sensors and perceptions algorithms to extract navigation information from unstructured environments. In [1], the problem of navigation in complex outdoor natural environments based on multi-sensor information was introduced presenting different type of terrain representations that were used for planning operations. New perception algorithms were also presented to facilitate the interpretation of the sensor information [2], [3], [4]. Nevertheless the solution of "Simultaneous Localization and Mapping" (SLAM) [7],[8] or "Concurrent Map and Localisation" (CML) [9] problem in large and totally unstructured environments presents formidable problems still unresolved.

The solution of SLAM has been addressed with approaches based on Bayesian filtering [10]. These techniques approximate the probability representation using samples of the probability density distribution. Although they are still computationally expensive for real time implementation they present significant advantages such as the inherent solution of the data association problem. There are several optimal and sub-optimal techniques that are attractive to solve SLAM in real time [5], [7], [9], [12], most of them based on the Extended Kalman Filter (EKF) framework, These techniques assume Gaussian or at least uni-modal probability density distributions.

It is well known that one of the major problems of EKF SLAM algorithms is the computational requirements that is of order $\sim O(N^2)$, being N the states used to represent the landmarks and vehicle pose. In [7], a Compressed EKF (CEKF) algorithm is presented that dramatically reduces the computational requirements of SLAM. This algorithm is very efficient when the vehicle remains in local areas for significant period of time or when high frequency external information is incorporated [6]. Still a full SLAM update is required when a transition to a different area is performed. Another important implementation aspect is the memory requirement of the SLAM algorithms to store the state covariance error matrix. A system with 10000 states will require up to 800 MB of RAM to maintain this matrix. Both memory and global update calculation have a cost of order $\sim O(N^2)$ as stated before. When using the CEKF the cost in the global update evaluation is not critical provided the transitions between local areas are not frequent but the memory requirements remain similar to the full SLAM implementation. This is due to the need of maintaining cross-covariance terms between all the states that have to be preserved. This implies that the complete covariance matrix has to be maintained in fast processing memory (RAM). We argue that the maintenance of the complete covariance matrix is important in cases where the cross-correlation between states is strong or at least not negligible. Any attempt to conservatively de-correlate a subset of states implies an increase in the value of some diagonal sub-matrixes. If subsets of lightly correlated states are present then de-correlation of these states can be done with a small loss in the predicted estimation quality. This paper makes use of RLR (Relative Landmark Representation) to generate close to optimal results that will significantly reduce the computational and memory requirement of SLAM when working in large environments.

This work is organized as follows. Section 2 presents a brief introduction to the CEKF filter. The decorrelation algorithm is presented in section 3 and the integration with the CEKF in SLAM applications is described in section 4. Finally experimental results are presented in section 5 with conclusions given in section 6.

1. Optimal Compressed Extended Kalman Filter (CEKF).

This section presents a brief summary of the CEKF algorithm. A full description is presented in [7]. Assume that for a period of time $\Omega = \{k / k_1 \le k \le k_2\}$, the model and observations of the system can be expressed in the following form:

$$
\begin{gathered}
X = \begin{bmatrix} X_a \\ X_b \end{bmatrix}, \quad X_a \in R^{N_a}, \quad X_b \in R^{N_b}, \\
\begin{bmatrix} X_a(k+1) \\ X_b(k+1) \end{bmatrix} = \begin{bmatrix} f_a\left(X_a(k), u(k), k\right) + v_a(k) \\ X_b(k) + v_b(k) \end{bmatrix} \\
y(k) = h\left(X_a(k), k\right) + v_h(k) \\
\forall (X, u, k) \quad / \quad k \in \Omega
\end{gathered}
\tag{1}
$$

Where X_a and X_b are active and passive states respectively and the model and observation noises $v_a(k), v_b(k), v_h(k)$ are Gaussian and uncorrelated. In this system the observations during the interval Ω are only function of the states X_a. In this case the states X_a can be estimated in a very efficient form using the CEKF algorithm as shown bellow. At the beginning of the period Ω, $k = k_1$, a set of auxiliary matrices are initialized:

$$
\begin{gathered}
\phi_{k_1} = I, \quad \psi_{k_1} = \bar{\bar{0}}, \quad \theta_{k_1} = \bar{0}, \quad Q^{*}_{bb,k_1} = \bar{\bar{0}} \\
\left(\phi_k, \psi_k \in R^{N_a x N_a}, \quad \theta_k \in R^{N_a}\right)
\end{gathered}
\tag{2}
$$

At every prediction or update stage, during the time period Ω, a normal EKF algorithm is run with the subsystem P_{aa}, $X_{a.}$, where P_{aa} is the covariance matrix error for the states $X_{a.}$ An additional set of matrices is maintained to store the information gathered to be transfered to the rest of the system when the full update stage is executed. During the period $k_1 \le k \le k_2$ the prediction step is implemented using the standard EKF prediction for the sub-system P_{aa}, X_a and the update of the auxiliary matrices:

$$\begin{gathered}\phi_k = J_{aa} \cdot \phi_{k-1}, \quad \psi_k = \psi_{k-1}, \quad \theta_k = \theta_{k-1} \\ \left(J_{aa} = \partial f_A \big/ \partial X_A \right) \\ Q^*_{bb,k} = Q^*_{bb,(k-1)} + Q_{bb,k}\end{gathered} \tag{3}$$

A new observation is processed with a standard EKF update equations for the sub-system P_{aa}, X_a and the update of the auxiliary matrices as follows:

$$\begin{gathered}\begin{matrix}\phi_k = (I - \mu_k) \cdot \phi_{k-1} \\ \psi_k = \psi_{k-1} + \phi^T_{k-1} \cdot \beta_k \cdot \phi_{k-1}\end{matrix} \left\{ \begin{matrix} H_{a,k} = \partial h \big/ \partial X_a \\ \beta_k = H^T_{a,k} \cdot S_k^{-1} \cdot H_{a,k} \\ \mu_k = P_{aa,k} \cdot \beta_k \end{matrix} \right. \\ \theta_k = \theta_{k-1} + \phi^T_{k-2} \cdot H^T_{a,k-1} \cdot S^{-1}_{k-1} \cdot z_{k-1}\end{gathered} \tag{4}$$

At the end of the interval Ω, $k = k_2$, the update of all the states in the system and covariance matrix is performed. This step is called global or full update:

$$\begin{gathered}P_{ab,(k_2)} = \phi_{(k_2)} \cdot P_{ab,(k_1)} \\ P_{bb,(k_2)} = P_{bb,(k_1)} - P_{ba,(k_1)} \cdot \psi_{(k_2)} \cdot P_{ab,(k_1)} + Q^*_{bb,k_2} \\ X_{b,(k_2)} = X_{b,(k_1)} - P_{ba,(k_1)} \cdot \theta_{k_2}\end{gathered} \tag{5}$$

It can be seen that the knowledge of $X_b(k), P_{bb}(k), P_{ab}(k), P_{ba}(k)$ is only explicit at times $k = k_1$ and $k = k_2$. No explicit information about this family of states and their related covariance and cross-covariance matrices is required at times $k / k_1 < k < k_2$. This information is implicitly contained (at any instant k) in the matrixes ϕ_k, ψ_k, θ_k. The matrices ϕ_k, ψ_k have dimensions $N_a * N_a$ and the vector θ_k is of order N_a. All the information (covariance and cross-covariance values) related to the states $X_b(k)$ remains compressed in these three auxiliary matrices. It is important to note that the CEKF estimation is optimal and generates the same results as the full EKF algorithm. Although these results are used here in a SLAM application the CEKF algorithm can be applied to any system where the observations remain function of a set of states for a period of time as shown in equation (1).

2 De-correlation algorithm

The maintenance of the complete covariance matrix is essential in cases where the cross-correlation between states is strong or at least not negligible. Any attempt to conservatively de-correlate a subset of states implies an increase in the value of some diagonal sub-matrixes. If subsets of weakly correlated states are present then de-correlation of these states can be done with a small loss in the predicted estimated quality. Unfortunately when a map represents the landmarks in absolute form with respect to a single global frame, all the states are or tend to be strongly correlated. In this case most of the state's high correlation coefficients are due to the map representation and not due to map estimation problem itself. This correlation is important when a conservative de-correlation procedure is desired.

An appropriate map representation that avoids this problem is the Relative Landmark Representation (RLR). This representation divides the map into sub-regions where the landmarks are defined respect to local coordinate frames [15]. For the 2-D case each local frame is defined based on two local landmarks represented in global coordinates. The high correlation characteristic persists but only between the frame base landmarks and the vehicle states. These landmarks represent a small subset of the total landmark population.

An attractive aspect of the RLR is that the cross-correlation between relative landmarks that belong to different frames (or constellations) tends to be extremely low, especially when these constellations are not adjacent or separated by a large distance. Figure 1 shows a typical cross correlation matrix obtained in an outdoor run. In this case grey levels from white to black are used to display the correlation coefficients. Each diagonal block corresponds to a different constellation. The bottom right block contains the vehicle and base landmarks and is cross correlated with the rest of the map. It can be seen that the cross-correlation terms between adjacent constellations are very weak and almost non-existent for distant constellation.

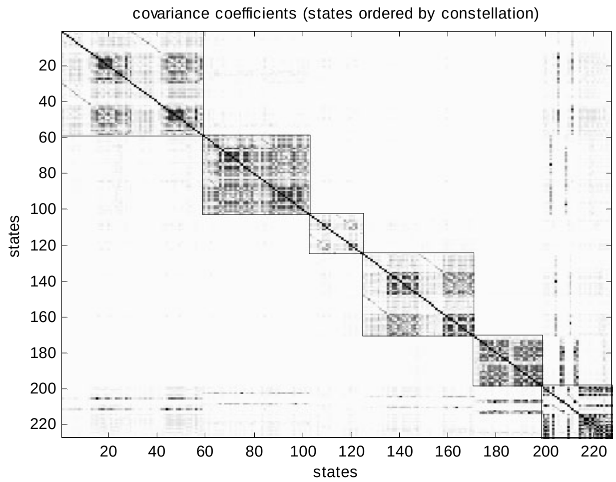

Figure 1 Correlation coefficients using a Relative Landmark Representation (RLR). The bottom right block has the vehicle state and all the absolute base landmarks. The other blocks have the relative states in each constellation. Cross-correlation is significant between the absolute states and the relative states and between relative states in the same constellation.

Although the off-diagonal terms are very close to zero they can not be eliminated without any modification of the diagonal blocks. Depending on the number of blocks a de-correlation approach can be designed by multiplying the diagonal terms by a factor greater than two. As will be seen later in this paper this approach will generate unnecessarily conservative results.

The previous discussion suggests that a much less conservative approach can be designed based on the off-diagonal terms. This approach will be very close to optimal since these terms are very close to zero making the increment of the diagonal terms very small as shown later in this section.

The algorithm to cancel the weakly cross-correlation terms in a consistent manner is now presented. Given a symmetric nonnegative definite matrix $P \geq 0, \quad P \in R^{2x2}$, it is possible to obtain a de-correlated (diagonal) matrix $D \geq P$ according to:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} =$$

$$= \begin{bmatrix} p_{11} + \kappa \cdot |p_{12}| & 0 \\ 0 & p_{22} + \frac{|p_{12}|}{\kappa} \end{bmatrix} - \begin{bmatrix} \kappa \cdot |p_{12}| & -p_{12} \\ -p_{12} & \frac{|p_{12}|}{\kappa} \end{bmatrix} = \qquad (6)$$

$$= D - \tau \leq \begin{bmatrix} p_{11} + \kappa \cdot |p_{12}| & 0 \\ 0 & p_{22} + \frac{|p_{12}|}{\kappa} \end{bmatrix} = D$$

$$\forall \quad \kappa > 0$$

This is true since τ is a nonnegative definite matrix:

$$\tau = \begin{bmatrix} \kappa \cdot |p_{12}| & -p_{12} \\ -p_{12} & \frac{1}{\kappa} \cdot |p_{12}| \end{bmatrix} \geq 0 \qquad \forall \quad \kappa > 0 \qquad (7)$$

It is always possible to de-correlate the covariance matrices corresponding to two groups of states α and β using a similar technique. Assuming a generic block matrix P represented as follow:

$$P = \begin{bmatrix} \alpha & C & D \\ C^T & \beta & E \\ D^T & E^T & \gamma \end{bmatrix}$$

$$\alpha \in R^{n\,x\,n} \quad , \quad \beta \in R^{m\,x\,m} \quad , \quad \gamma \in R^{l\,x\,l} \quad , \quad C \in R^{n\,x\,m}, \ldots (8)$$

$$C = \begin{bmatrix} c_{11} & .. & .. & c_{1m} \\ .. & .. & .. & .. \\ .. & .. & .. & .. \\ c_{n1} & .. & .. & c_{nm} \end{bmatrix}$$

It can be partially de-correlated as

$$\begin{bmatrix} \alpha & C \\ C^T & \beta \end{bmatrix} = \begin{bmatrix} \alpha & \bar{\bar{0}} \\ \bar{\bar{0}} & \beta \end{bmatrix} - \begin{bmatrix} \tilde{\alpha} & -C \\ -C^T & \tilde{\beta} \end{bmatrix} + \begin{bmatrix} \tilde{\alpha} & 0 \\ 0 & \tilde{\beta} \end{bmatrix} =$$

$$= \begin{bmatrix} \alpha + \tilde{\alpha} & 0 \\ 0 & \beta + \tilde{\beta} \end{bmatrix} - \begin{bmatrix} \tilde{\alpha} & -C \\ -C^T & \tilde{\beta} \end{bmatrix} \leq \begin{bmatrix} \alpha + \tilde{\alpha} & 0 \\ 0 & \beta + \tilde{\beta} \end{bmatrix} \qquad (9)$$

$$\tilde{\alpha}, \tilde{\beta} \quad / \quad \begin{bmatrix} \tilde{\alpha} & -C \\ -C^T & \tilde{\beta} \end{bmatrix} \geq 0$$

$$\Downarrow$$

$$P = \begin{bmatrix} \alpha & C & D \\ C^T & \beta & E \\ D^T & E^T & \gamma \end{bmatrix} \leq \begin{bmatrix} \alpha + \tilde{\alpha} & \bar{\bar{0}} & D \\ \bar{\bar{0}} & \beta + \tilde{\beta} & E \\ D^T & E^T & \gamma \end{bmatrix}$$

The matrix *P* will be nonnegative definite if the matrices $\tilde{\alpha}$ and $\tilde{\beta}$ are formed with the following expressions:

$$\tilde{\alpha} \quad / \quad \tilde{\alpha}_{i,j} = \begin{cases} \sum_{k=1}^{m} \kappa_{i,k} \cdot \left| c_{i,k} \right| & , \quad i = j \\ 0 & , \quad i \neq j \end{cases}$$

$$\tilde{\beta} / \tilde{\beta}_{i,j} = \begin{cases} \sum_{k=1}^{n} \tilde{\kappa}_{i,k} \cdot \left| \tilde{c}_{i,k} \right| = \sum_{k=1}^{n} \frac{1}{\kappa_{k,i}} \cdot \left| c_{k,i} \right| & , \quad i = j \\ 0 & , \quad i \neq j \end{cases} \tag{10}$$

$$\kappa_{i,k} > 0 \quad \forall \quad i,k$$

Equation 10 guarantees that the matrix *P* will be, at least, nonnegative definite.

Selecting $\kappa_{i,k} = \tilde{\kappa}_{k,i} = 1$ the coefficients α and β becomes:

$$\tilde{\alpha}_{i,i} = \sum_{k=1}^{m} \left| c_{i,k} \right|, \quad \tilde{\beta}_{j,j} = \sum_{k=1}^{n} \left| c_{k,j} \right| \tag{11}$$

A less conservative selection of the family $\left\{ \kappa_{i,k} \right\}$ can be done considering the cross-correlations coefficients:

$$\kappa_{i,k} = \sqrt{\frac{\alpha_{i,i}}{\beta_{k,k}}} = \frac{\alpha_{i,i}}{\sqrt{\alpha_{i,i} \cdot \beta_{k,k}}}$$

$$\tilde{\kappa}_{k,i} = \frac{1}{\kappa_{i,k}} = \sqrt{\frac{\beta_{k,k}}{\alpha_{i,i}}} = \frac{\beta_{k,k}}{\sqrt{\alpha_{i,i} \cdot \beta_{k,k}}} \tag{12}$$

Then α and β are evaluated:

$$\tilde{\alpha}_{i,i} = \sum_{k=1}^{m} \left| c_{i,k} \right| \cdot \kappa_{i,k} = \alpha_{i,i} \cdot \sum_{k=1}^{m} \left| c_{i,k} \right| \cdot \frac{1}{\sqrt{\alpha_{i,i} \cdot \beta_{k,k}}}$$

$$\tilde{\beta}_{j,j} = \sum_{k=1}^{n} \left| \tilde{c}_{j,k} \right| \cdot \tilde{\kappa}_{j,k} = \sum_{k=1}^{n} \left| c_{k,j} \right| \cdot \frac{\beta_{j,j}}{\sqrt{\alpha_{k,k} \cdot \beta_{j,j}}} = \tag{13}$$

$$= \beta_{j,j} \cdot \sum_{k=1}^{n} \left| c_{k,j} \right| \cdot \frac{1}{\sqrt{\alpha_{k,k} \cdot \beta_{j,j}}}$$

Finally the diagonal coefficients are updated:

$$\begin{aligned}\alpha_{i,i} + \tilde{\alpha}_{i,i} &= \alpha_{i,i} \cdot \left(1 + \sum_{k=1}^{m} \left|\mu_{i,k}\right|\right) \\ \beta_{i,i} + \tilde{\beta}_{i,i} &= \beta_{i,i} \cdot \left(1 + \sum_{k=1}^{n} \left|\mu_{k,i}\right|\right)\end{aligned} \tag{14}$$

with $\mu_{i,k} = \frac{c_{i,k}}{\sqrt{a_{i,i} \cdot b_{k,k}}}$ (15)

If the states to be de-correlated have very low correlation then the correction terms $\sum_{k=1}^{m} \left|\mu_{i,k}\right|$ and $\sum_{k=1}^{n} \left|\mu_{k,i}\right|$ will be small. Since the RLR generate this type of matrix, the increment in the diagonal terms will be very small making the de-correlation approach very close to optimal.

3 Sub-Optimal CEKF SLAM

This section presents the integration of the de-correlation and the CEKF algorithm. The following aasumptions are made:

1. The landmark map is created using a Relative Landmark Representation (RLR).
2. All correlations between states will be maintained except the correlations between relative landmarks that belong to distant constellations. These states will be de-correlated using techniques presented in the previous section.
3. After each CEKF global update the generated cross-correlation between relative landmarks of different constellations will be small making the de-correlation strategy close to optimal.
4. The states are partitioned into active and passive states according to the proximity of the vehicle and the type of landmark, i.e absolute or relative.

Any landmark that is a base frame landmark is called ‘absolute landmark’ since it is represented in global coordinates. A landmark that is represented in a local frame is named ‘relative landmark’.

A state is called ‘absolute state’ if it is related to the vehicle kinematics or to an absolute landmark. Any state associated to a relative landmark is called ‘relative state’.

The assumptions (1) and (3) are strictly related. If the RLR map representation is used then the correlation between the states representing the relative landmarks that belong to different constellations tends to be very small. Conversely, the correlation between absolute states tends to be strong especially between states representing close absolute landmarks. It is then possible to design an algorithm to preserve the cross-correlation of any absolute state with any other state (relative or absolute) and to ignore (de-correlate) any cross-correlation between two relative states associated to two relative landmarks that belong to different constellations (defined in different local frames). The approach proposed also preserves the cross-correlation terms between relative landmarks of the same constellation (or close constellations).

A normal EKF full SLAM performing this de-correlation in each update will result in excessively conservative results since over-bounding will be required in each update to de-correlate. Conversely, a CEKF will only require the full update after many local updates rendering in a less conservative over-bounding strategy. In the CEKF algorithm a global update is required only when the vehicle abandons a sub-region.

The landmarks are divided in different categories according to their location with respect to the actual position of the vehicle. Figure 2 shows the vehicle navigating in an area with various types of landmarks. The triangles are base landmarks and the circles represent relative landmarks measured with respect to their own base. For 2-D navigation two landmarks are required to define a base. The active landmarks X_a are formed with all the relative (X_{aR}) and base landmarks (X_{aB}) in the local area and the base landmarks that have relative landmarks in the local area. The relative and absolute landmarks are shown in Figure 2 as black circles and triangles respectively.

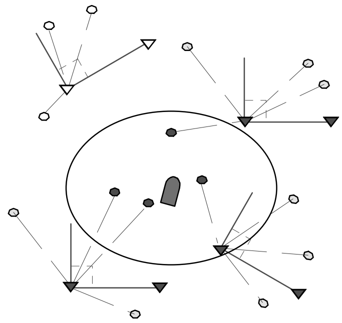

Figure 2 Active and passive landmarks. The active landmarks are the relative landmarks in the local area (black circles) and the absolute landmarks in the local area plus the ones that have an active relative landmark in the local area (black triangles). Finally the white circles and triangles represent far relative and base landmarks.

The passive states group (X_b) is formed with the relative landmarks states of the constellations where the vehicle is not navigating in and all other base landmarks.

The states of the passive relative landmarks can then be divided in two groups:

1) X_{bR_1} , Relative landmarks that belong to the same constellation of active relative landmarks or to adjacent constellations.
2) X_{bR_2} , Relative landmarks that belong to distant constellations.

The full covariance matrix representing the error in the states will have the form presented in Figure 3

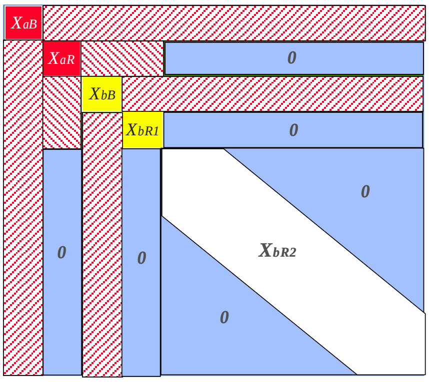

Figure 3 State error covariance matrix. The full correlation between the absolute states X_{aB} and X_{bB} and the rest of the states is always maintained. The relative state cross-correlation is only maintained between same and close constellations.

The cross-correlations between states X_{bR_1} with X_{bR_2} and X_{aR} with X_{bR_2} are set to zero with appropriate modification of the terms in X_{bR_1} and X_{aR} according to the de-correlation procedures presented in section 3. It can be seen that the only elements that needs to be maintained are contained in a band matrix of reduced size. The cross-correlation with the absolute states is also maintained. The relative sizes of the different matrices are shown in Figure 4 where it can be seen that the majority of elements are not needed to be maintained. The filter runs as an optimal CEKF except when a global update needs to be performed due to a region transition.

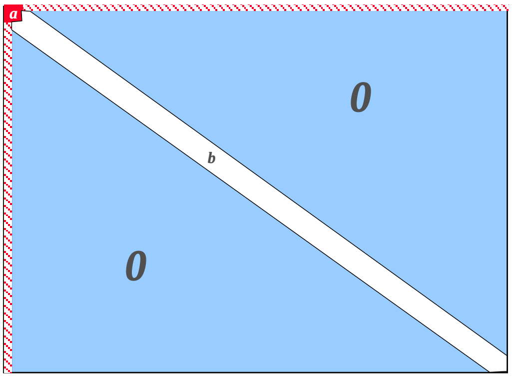

Figure 4 Relative matrix's sizes in a standard application. The width of the "b" matrix is function of the number of of landmarks in the vicinity of the vehicle.

In practice most of the information obtained in the local area is transmitted to the state corresponding to the vehicle pose, base landmarks and local relative landmarks states. This means that the improvement in the passive states covariance sub-matrix can be ignored without loosing significant information. This procedure has a similar effect of adding some uncertainty to the passive states after the CEKF global update is done. Independently of if the change in the passive states covariance sub-matrix is ignored or not, the cross-covariance between active and passive states has to be updated if no de-correlation is applied. Then the de-correlation will act over the relative states block of the active-passive cross-correlation sub-matrix terms.

With this approach the memory and computation requirements will be $\sim O(N^*N_b)$, assuming a constant number of landmarks N_b is used. Since N_b is << N the computation and memory requirements of the algorithm are dramatically reduced.

The implementation of the strategy proceeds as follows: The complete optimal global update of the passive states is done after r CEKF internal steps:

$$\begin{aligned} k_2 &= k_1 + r \\ P_{ab,(k_2)} &= \phi_{(k_2)} \cdot P_{ab,(k_1)} \\ P_{bb,(k_2)} &= P_{bb,(k_1)} - P_{ba,(k_1)} \cdot \psi_{(k_2)} \cdot P_{ab,(k_1)} \\ X_{b,(k_2)} &= X_{b,(k_1)} - P_{ba,(k_1)} \cdot \theta_{k_2} \end{aligned} \quad (16)$$

With the sub-optimal CEKF approach most of the improvement of P_{bb} is ignored. The CEKF auxiliary matrix ψ is still needed to evaluate the part of P_{bb} not ignored. The objective is to transfer the new information to the states representing the vehicle pose, absolute landmarks and local relative landmarks, ignoring completely the covariance changes in the non-local relative landmarks states. From the viewpoint of the non-local relative landmarks, no change in their quality is obtained.

In practical SLAM applications it can be observed that the cross-correlation factors between relative landmarks of different constellations have values of order 10^{-4} or smaller. This value becomes much smaller for distant constellations. This characteristic makes the conservative de-correlation close to optimal since very small virtual noise has to be added to the relative landmarks covariances to obtain the de-correlated matrix bounds.

A less aggressive de-correlation strategy can be implemented by accepting the existence of cross-correlation between relative landmarks of different constellations provided that these constellations are geographically close. Then de-correlation is implemented only between distant constellations. This will increase the width of the band matrix in Figure 4 . In the particular case when a cross-correlation factor is not small enough one of the involved states can be degraded to quality zero.

A final comment on the consistency of the method is noteworthy. It is well known that any Kalman Filter based system is prone to catastrophic failures under a data association problem. This could be due to failure in the feature extraction algorithm, or in the observation or prediction models. Under this situation it is very difficult to guarantee the consistency of the algorithm. These types of problems are common in difficult environments such as underwater [9],[14], where vehicle, models and sensor information have significant uncertainty. In many other applications, such as the ones related to land vehicles, it is possible to have combinations of vehicle models and sensors to provide the level of integrity required to detect any possible data association problem if ever occurs. In such cases the EKF become the most efficient tool to solve the SLAM problem. This work presents consistent and close to optimal solutions that are applicable to these common types of applications.

4 Results

The algorithm was implemented using a data set logged with the utility vehicle shown in Figure 5. The vehicle is retrofitted with velocity and steering encoders, two lasers range sensors and GPS. A Compass sensor was not used since the density of landmark in the environment was enough to maintain low heading errors. The GPS is used to obtain ground truth. The vehicle operated in a large outdoor unstructured environment similar to the one shown in Figure 5.

Figure 5. Utility vehicle and outdoor environment. The vehicle is equipped with laser range sensors, steering and velocity encoders.

The final trajectory and map using the Full SLAM and the sub-optimal CEKF are superimposed in Figure 6. In this case an aggressive de-correlation policy was used. The filter conservatively ignores the cross-correlation between relative states that belong to different constellations. In this case a band matrix of approximate size of $N*N_a$ will be maintained. N_a is the maximum number of landmarks in a local area. For this particular case the covariance matrix has a total size of approximately 250000 elements. With the algorithm proposed the memory required is less than 15000 elements. A less conservative approach will consider more than one consecutive constellation to maintain the cross-correlation between relative landmarks of adjacent constellations. This will require additional memory to maintain the relevant coefficients but will still will be significant smaller that maintaining the full covariance matrix. For larger system the improvement will be more significant.

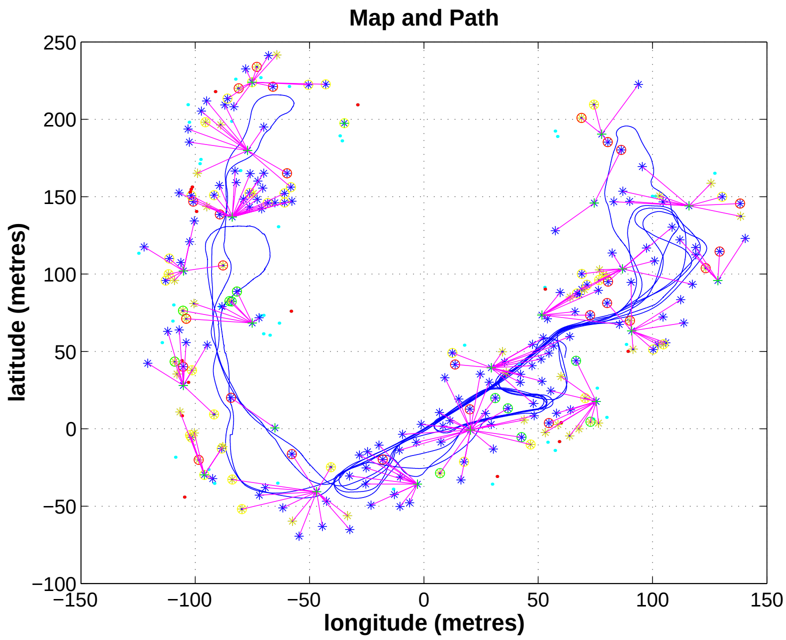

Figure 6. Final Trajectory and Map. The figure presents the superposition of the final trajectory generated by the full EKF and the CEKF with the de-correlation techniques presented in this paper. All the landmarks that belong to a constellation are joined with lines to a base. Although an aggressive de-correlation was utilized (elimination of cross-correlations of adjacent constellation) the difference with the full EKF is not noticeable using this scale.

Figures 7 and 8 present the position and heading difference between the optimal and sub-optimal approaches. It can be seen that the maximum discrepancy is smaller than 0.6 meters and 0.3 degrees respectively.

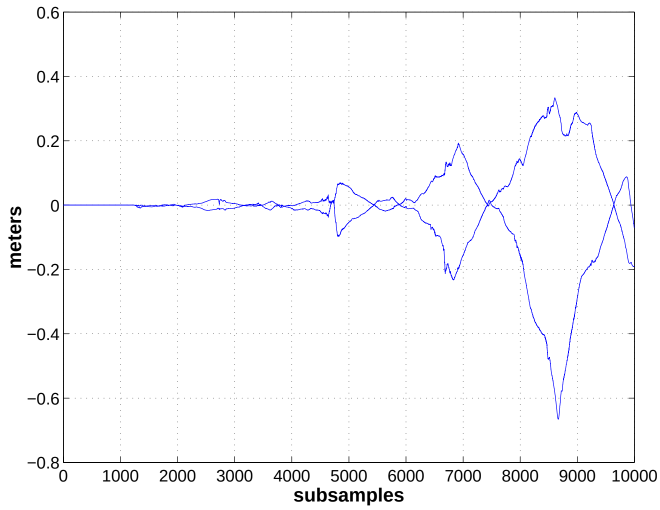

Figure 7 Difference between full SLAM and conservative approach for latitude and longitude estimation. The maximum discrepancy of the algorithm using the aggressive de-correlation is less than 0.6 meter in the worst case.

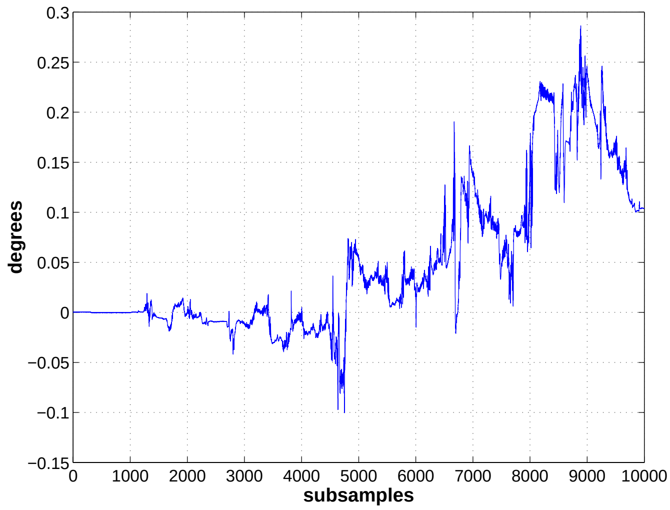

Figure 8 Difference between full SLAM and conservative approach for vehicle heading estimation. The maximum discrepancy of the algorithm using the aggressive de-correlation is less than 0.3 degree in the worst case.

Finally Figure 9 presents the correlation coefficient matrix. A black region represents the elements where the correlation was forced to zero. It can be seen that a significant part of the matrix does not need to be maintained. It is noteworthy to explain the difference with respect to Figure 2 . Each diagonal block corresponds to different constellations. Each block has at two base states that are cross-correlated to the rest of the states as expected. These are the only cross correlations that need to be maintained. In Figure 2 all the base and vehicle states are displayed as part of the bottom right block sub-matrix. In Figure 9 the states are ordered according to the time they were incorporated as new landmarks. It can also be appreciated the significant saving in term of memory since most of elements of the matrix are zero and do not need to be stored.

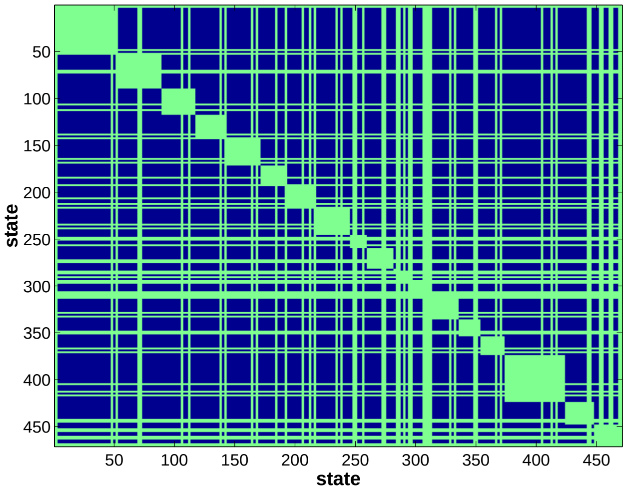

Figure 9 Correlation coefficient matrix. Each block corresponds to a constellation formed by two base landmarks and various relative landmarks. The base landmarks are strongly correlated to the rest of the system. The black elements correspond to the coefficients that were set to zero. Most of the elements of this matrix are very close to zero.

5 Conclusions

The Compressed EKF (CEKF) algorithm reduces the computational requirements of SLAM to the order of $O\sim(N_a^2)$ when navigating in the local area requiring a full update of $O\sim(N^2)$ each time a transition to a different area is performed. In this case N is proportional to the number of features incorporated to the map and N_a proportional to the states representing the landmarks in the local area. This paper presents new algorithms to reduce the computational requirement of the full SLAM update to approximately $O\sim(N*N_b)$, with N_b the states representing the landmarks in the active/local states. With this implementation the memory requirements are also reduced to order $N*N_b$. Since N_b is << N the computation and memory requirements of the algorithm are dramatically reduced. The experimental results have also demonstrated that by using the appropriate map representation the results obtained are very close to optimal as expected from the derivation presented in section 3.

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