The following is some working for my paper on FastSLAM consistency. In particular, notice the derivation of Equ. 8, which is the likelihood function for the particle states. It is dependent on the vehicle and observation histories, but *not* the map. Tim Bailey

The efficiency of the FastSLAM algorithm depends on a Rao-Blackwellised implementation of the particle filter, which requires a partitioning of the SLAM posterior into vehicle and map components as follows.

$$\begin{aligned} p\left(\mathbf{X}_{v_{0:k}}, \mathbf{m} | \mathbf{Z}_{0:k}, \mathbf{U}_{0:k}\right) \\ = p\left(\mathbf{X}_{v_{0:k}} | \mathbf{Z}_{0:k}, \mathbf{U}_{0:k}\right) p\left(\mathbf{m} | \mathbf{X}_{v_{0:k}}, \mathbf{Z}_{0:k}, \mathbf{U}_{0:k}\right) \\ = p\left(\mathbf{X}_{v_{0:k}} | \mathbf{Z}_{0:k}, \mathbf{U}_{0:k}\right) p\left(\mathbf{m} | \mathbf{X}_{v_{0:k}}, \mathbf{Z}_{0:k}\right) \end{aligned} \tag{1}$$

This gives a vehicle part $p\left(\mathbf{X}_{v_{0:k}} | \mathbf{Z}_{0:k}, \mathbf{U}_{0:k}\right)$ which evolves according to non-linear process and observation models, and a map part $p\left(\mathbf{m} | \mathbf{X}_{v_{0:k}}, \mathbf{Z}_{0:k}\right)$ which has a trivial process model, and a non-linear observation model. Notice that the SLAM PDF is defined here in terms of the vehicle pose history $\mathbf{X}_{v_{0:k}}$. This is necessary to derive the efficient map representation that gives FastSLAM its $O(N)$ computational properties. Conditioned on the pose history, the individual landmarks are independent

$$p\left(\mathbf{m} | \mathbf{X}_{v_{0:k}}, \mathbf{Z}_{0:k}\right) = \prod_{i=1}^{N} p\left(\mathbf{m}_i | \mathbf{X}_{v_{0:k}}, \mathbf{Z}_{i_{0:k}}\right) \tag{2}$$

where $\mathbf{Z}_{i_{0:k}}$ is the set of observations of landmark $\mathbf{m}_i$. The dependence on $\mathbf{X}_{v_{0:k}}$ provides the key advantage of FastSLAM and, at the same time, is its primary weakness.

We address first the map part of Equ. 1. According to Bayes theorm, the map update can be expressed as

$$\begin{aligned} p\left(\mathbf{m} | \mathbf{X}_{v_{0:k}}, \mathbf{Z}_{0:k}\right) \\ = \frac{p\left(\mathbf{z}_k | \mathbf{X}_{v_{0:k}}, \mathbf{m}, \mathbf{Z}_{0:k-1}\right) p\left(\mathbf{m} | \mathbf{X}_{v_{0:k}}, \mathbf{Z}_{0:k-1}\right)}{p\left(\mathbf{z}_k | \mathbf{X}_{v_{0:k}}, \mathbf{Z}_{0:k-1}\right)} \\ \propto p\left(\mathbf{z}_k | \mathbf{X}_{v_{0:k}}, \mathbf{m}, \mathbf{Z}_{0:k-1}\right) p\left(\mathbf{m} | \mathbf{X}_{v_{0:k}}, \mathbf{Z}_{0:k-1}\right) \end{aligned} \tag{3}$$

Given the assumption that the observations are independent conditioned on the state,

$$p\left(\mathbf{Z}_{0:K} | \mathbf{X}_{v_{0:K}}, \mathbf{m}\right) = \prod_{k=0}^{K} p\left(\mathbf{z}_k | \mathbf{x}_{v_k}, \mathbf{m}\right) \tag{4}$$

the map update becomes

$$p\left(\mathbf{m} | \mathbf{X}_{v_{0:k}}, \mathbf{Z}_{0:k}\right) \propto p\left(\mathbf{z}_k | \mathbf{x}_{v_k}, \mathbf{m}\right) p\left(\mathbf{m} | \mathbf{X}_{v_{0:k-1}}, \mathbf{Z}_{0:k-1}\right) \tag{5}$$

The vehicle part of Equ. 1 is a dynamic system governed by a predict step and an update step. The transition prior is obtained as

$$\begin{aligned} p\left(\mathbf{X}_{v_{0:k}} | \mathbf{Z}_{0:k-1}, \mathbf{U}_{0:k}\right) = \\ p\left(\mathbf{x}_{v_k} | \mathbf{x}_{v_{k-1}}, \mathbf{u}_k\right) p\left(\mathbf{X}_{v_{0:k-1}} | \mathbf{Z}_{0:k-1}, \mathbf{U}_{0:k-1}\right) \end{aligned} \tag{6}$$

Notice that this predict step does not involve the Chapman-Kolomogorov equation, since the previous state $\mathbf{x}_{v_{k-1}}$ is not marginalised away. The update step is given by

$$\begin{aligned} p\left(\mathbf{X}_{v_{0:k}} | \mathbf{Z}_{0:k}, \mathbf{U}_{0:k}\right) \propto \\ p\left(\mathbf{z}_k | \mathbf{X}_{v_{0:k}}, \mathbf{Z}_{0:k-1}\right) p\left(\mathbf{X}_{v_{0:k}} | \mathbf{Z}_{0:k-1}, \mathbf{U}_{0:k}\right) \end{aligned} \tag{7}$$

where the likelihood function is found by marginalising over the landmarks

$$\begin{aligned} p\left(\mathbf{z}_k | \mathbf{X}_{v_{0:k}}, \mathbf{Z}_{0:k-1}\right) \\ = \int p\left(\mathbf{z}_k | \mathbf{x}_{v_k}, \mathbf{m}\right) p\left(\mathbf{m} | \mathbf{X}_{v_{0:k-1}}, \mathbf{Z}_{0:k-1}\right) d\mathbf{m} \end{aligned} \tag{8}$$

Notice that, due to marginalisation over map, the likelihood is now dependent on the state and observation histories. Combining Equs. 6 and 7, a recursive expression for the vehicle pose history is

$$\begin{aligned} p\left(\mathbf{X}_{v_{0:k}} | \mathbf{Z}_{0:k}, \mathbf{U}_{0:k}\right) \propto \\ p\left(\mathbf{z}_k | \mathbf{X}_{v_{0:k}}, \mathbf{Z}_{0:k-1}\right) p\left(\mathbf{x}_{v_k} | \mathbf{x}_{v_{k-1}}, \mathbf{u}_k\right) \\ p\left(\mathbf{X}_{v_{0:k-1}} | \mathbf{Z}_{0:k-1}, \mathbf{U}_{0:k-1}\right) \end{aligned} \tag{9}$$